Exam 2 F 2006.

Part 1, MULTIPLE CHOICE, 5 Points Each

1 An experiment consists of drawing a letter at random from the collection

$$U = \{w, e, l, u, v, m, a, t, h\}.$$

Let E be the event "A vowel is drawn" and F the event "a t or an h is drawn". Then $F \cup E'$ corresponds to the event:

(a) $\emptyset$ (b) $\{h, a, m, l, e, t\}$ ~~(c)~~ $\{t, h, m, l, v, w\}$ (d) $\{m, l, v, w\}$ (e) $\{t, h\}$

$E = \{e, u, a\}$ $E' = \{w, l, v, m, t, h\}$.

$F = \{t, h\}$

$F \cup E' = \{w, l, v, m, t, h\} \cup \{t, h\}$.

$= \{w, l, v, m, t, h\}$.

2 An experiment has five outcomes in its sample space, $\{s_1, s_2, s_3, s_4, s_5\}$. If

$$Pr(s_1) = 0.2, \quad Pr(s_2) = 0.3, \quad Pr(s_3) = 0.1, \quad Pr(s_4) = 0.2$$

then which of the following corresponds to $Pr(s_5)$?

(a) 1 ~~(b)~~ 0.2 (c) 0.8 (d) 0 (e) 0.5

Out	Prob.
s_1	.2
s_2	.3
s_3	.1
s_4	.2
s_5	?
	1

$Pr(s_5) = 1 - (.2 + .3 + .1 + .2)$

$= 1 - .8 = .2$.

3 Which of the following gives the coefficient of x^3y^5 in the binomial expansion of $(x+y)^8$?

(*a*) $P(8,3)$ (*b*) $P(5,3)$ (*c*) $C(5,3)$ ~~(*d*)~~ $C(8,3)$ (*e*) 1

$n = 8$

$(x+y)^n = C(n,0)x^0y^n + C(n,1)x^1y^{n-1} + \dots$

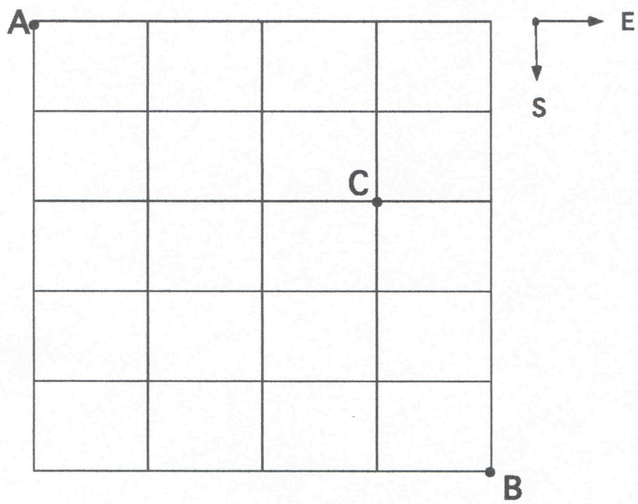
Terms are $C(n,k)\,x^k y^{n-k}$

match with $x^3 y^5$

To get coefficient $C(n,3)$

$= C(8,3) = (d)$

4 The drawing below shows a map of the streets in Joggerville. A jogger starts at point A and selects, at random, a path to point B, always running south or east. What is the probability that he **does Not** pass through point C?



(*a*) $\dfrac{C(5,3)\cdot C(4,1)}{C(9,4)} - 1$ (*b*) $\dfrac{1}{C(9,4)}$ (*c*) $\dfrac{5\cdot 4}{C(9,4)}$

~~(*d*)~~ $1 - \dfrac{C(5,3)\cdot C(4,1)}{C(9,4)}$ (*e*) $\dfrac{C(5,3)\cdot C(4,1)}{C(9,4)}$

Pr(Does NOT RUN THRU C) = ~~#P...~~

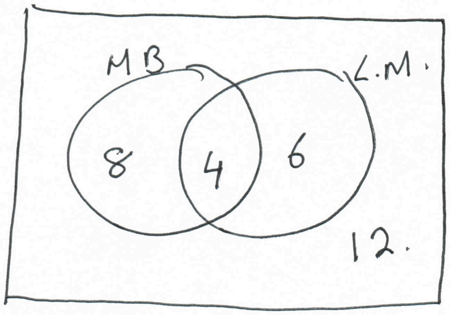
$= 1 - P(\text{Does RUN THRU C}) = 1 - \dfrac{\#\text{ Paths thru C}}{\text{Total }\#\text{ Paths}}$

$= 1 - \dfrac{(\#\text{ways from }A \to C)\cdot(\#\text{ ways from }C \to B)}{\text{Total }\#\text{ ways }A \to B} = 1 - \dfrac{C(5,3)\,C(4,1)}{C(9,4)}$

[illegible] $= (d)$

5 In a class (of 30 students) on French literature , 12 of the students have read "Madame Bovary", 10 have read "Les Miserables" and 4 have read both of these novels. What is the probability that a randomly selected student from the class has read "Madame Bovary" **given** that the student has read "Les Miserables"?

$(a)\ \frac{2}{5}$ $\quad (b)\ \frac{5}{9}$ $\quad (c)\ \frac{2}{9}$ $\quad (d)\ \frac{4}{9}$ $\quad \cancel{(e)}\ \frac{1}{3}$



$n(U)=30$

$$P(LM|MB) = \frac{P(LM \cap MB)}{P(MB)}$$

$$= \frac{\frac{(\# LM \cap MB)}{30}}{\frac{\# MB}{30}} = \frac{4}{12} = \frac{1}{3}$$

6 Let E and F be events associated with the same experiment. Suppose that E and F are **independent** and that $Pr(E) = \frac{1}{4}$ and $Pr(F) = \frac{1}{2}$. Then $Pr(E \cup F) =$

$(a)\ \frac{1}{8}$ $\quad (b)\ \frac{3}{4}$ $\quad (c)\ 0$ $\quad (d)\ \frac{7}{8}$ $\quad \cancel{(e)}\ \frac{5}{8}$

E and F independent $\Rightarrow$ $Pr(E \cap F) = Pr(E) \cdot Pr(F)$

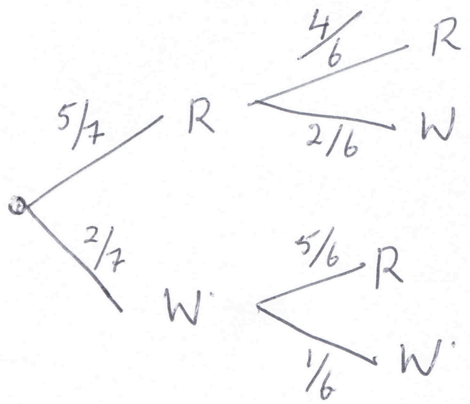
$$= \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$$

$$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} = \frac{5}{8}.$$

7 An urn contains 5 red balls and 2 white balls. A person has been instructed to select a ball at random from the urn, record its color, and then, without replacing the first ball, select a second ball from the urn and record its color. What is the probability that the second ball drawn is white? (Hint: Use a tree diagram)

~~(a)~~ $\frac{12}{42}$ (b) $\frac{3}{42}$ (c) $\frac{30}{42}$ (d) $\frac{10}{42}$ (e) $\frac{2}{6}$



$P(2^{nd}\,W)$

$= P(RW) + P(WW)$

$= \frac{5}{7}\cdot\frac{2}{6} + \frac{2}{7}\cdot\frac{1}{6}$

$= \frac{12}{42}$

8 A student (Richard Risky) is taking a multiple choice exam with 10 multiple choice questions. Richard, who didn't study because his pet alligator ate his book and homework, takes a random guess for each question. Each question has 5 choices for the answer (labeled (a), (b), (c), (d) and (e)). What is the probability that Richard will get at least 3 questions correct?

(a) $C(10,3)(.2)^3(.8)^7$

(b) $(.8)^{10} + C(10,1)(.2)^1(.8)^9 + C(10,2)(.2)^2(.8)^8$

~~(c)~~ $1 - \left\{ (.8)^{10} + C(10,1)(.2)^1(.8)^9 + C(10,2)(.2)^2(.8)^8 \right\}$

(d) $1 - C(10,3)(.2)^3(.8)^7$

(e) $1 - (.8)^{10}$

Binomial

n trials = 10 Trials

p = prob. success on each trial

= .2

X = # success = # correct.

$P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - (P(X=0) + P(X=1) + P(X=2))$

$= 1 - ((.8)^{10} + C(10,1)(.2)^1(.8)^9 + C(10,2)(.2)^2(.8)^8)$

9 The frequency distribution for the scores for a project given to a math class are recorded in the following table:

score	10	9	8	7	6	5	4	
frequencies	9	10	1	5	2	2	1	$N=30$

What is the mean for the above scores(that is the average of the scores)?

(*a*) 8 (*b*) 249 (*c*) 7 ~~(*d*)~~. 8.3 (*e*) 7.9

$$\frac{\sum f_i \cdot x_i}{N} = \quad \bar{X} = \frac{9\cdot 10 + 10\cdot 9 + 1\cdot 8 + 5\cdot 7 + 2\cdot 6 + 2\cdot 5 + 1\cdot 4}{30}$$

$$= \frac{249}{30} = 8.3$$

10 The probability distribution for a random variable X is given below? What is the probability distribution for the random variable $Z = X - 1$.

k	$\Pr(X = k)$	$X-1$ $k-1$
0	.3	-1
1	.2	0
2	.1	1
3	.1	2
4	.3	3

(*a*)

k	$\Pr(Z = k)$
0	.3
1	.2
2	.1
3	.1
4	.3

(*b*)

k	$\Pr(Z = k)$
0	$-.7$
1	$-.8$
2	$-.9$
3	$-.9$
4	$-.7$

~~(*c*)~~

k	$\Pr(Z = k)$
-1	.3
0	.2
1	.1
2	.1
3	.3

(*d*)

k	$\Pr(Z = k)$
0	.6
1	.4
2	.2
3	.2
4	.6

(*e*)

k	$\Pr(Z = k)$
1	.3
2	.2
3	.1
4	.1
5	.3

Part II, PARTIAL CREDIT,

Show all of your work for credit

11, 10 points A pair of dice (one red and one green) are rolled and the numbers on the uppermost faces are recorded. Consider the events:

E: the sum of the two numbers is 11. $\{(5,6)\ (6,5)\}$.

F: At least one number is odd. $36 - \#G = \#F$.

G : Both numbers are even. $\{(2,2)\ (4,4)\ (6,6)\ (2,4)\ (2,6)\ (4,2)\ (4,6)\ (6,2)\ (6,4)\}$.

(a) How many outcomes are in the sample space for this experiment?

$6 \times 6 = 36$ pairs

(b) Are E and G mutually exclusive events? Give a reason for your answer.

$E \cap F = \{(5,6)\ (6,5)\}$.

So E and F are Not Mutually Exclusive.

(c) Evaluate $Pr(G)$ and $Pr(F)$.

$$Pr\,G = \frac{\#G}{\#S.S.} = \frac{3 \times 3}{36} = \frac{9}{36} = \frac{1}{4}.$$

$$Pr(F) = 1 - Pr(G)$$

$F = G'$

$$= 1 - 1/4 = 3/4.$$

(d) Write down the set of outcomes corresponding to $E \cap F$.

$E \cap F = \{(5,6)\ (6,5)\}$.

12, 10 Points Let E and F be events in a sample space with $Pr(E) = 0.4$, $Pr(F) = 0.6$ and $Pr(E \cap F) = 0.3$.

(a) Are E and F independent events? Give a reason for your answer.

Is $P(E \cap F) = Pr(E) \cdot Pr(F)$

is $.3 = (.4)(.6)$

is $.3 = .24$.

(b) Calculate $Pr(E|F)$.

$$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)} = \frac{.3}{.6} = \frac{1}{2}.$$

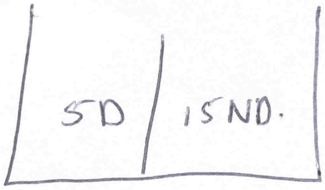
(c) Calculate $Pr(F|E)$.

$$Pr(F|E) = \frac{Pr(E \cap F)}{Pr(E)} = \frac{.3}{.4} = \frac{3}{4}.$$

(d) Calculate $Pr(E \cup F)$. $= Pr(E) + Pr(F) - Pr(E \cap F)$

$= .4 + .6 - .3 = .7.$

13, 10 points (a) A factory produces lightbulbs, which are packaged in boxes of 20. The quality control inspector selects a sample of 3 lightbulbs from each box. If he finds at least one defective lightbulb, the box is rejected and sent back to the factory. Otherwise the box is shipped. If a box contains 5 defective bulbs, what are the chances that it will be shipped?



Sample Size 3

$$Pr(\text{No Defectives}) = \frac{\#\text{ Samples with No Defectives}}{\text{Total }\#\text{ Samples.}}$$

$$= \frac{C(15,3)}{C(20,3)} \simeq .399$$

(b) Each of the lightbulbs from the above factory has probability 0.2 of failure in the first 700 hours of use. Suppose you use three of them to light your dorm room, and leave town for 700 hours (about a month). Assuming that the failures of the lightbulbs are independent of each other, what is the probability that at least one of the bulbs will still be lit when you return?

$Pr(F) = .2.$

$Pr(\text{at least one lit})$

$= 1 - Pr(\text{all Fail})$

$= 1 - ((.2).(.2)(.2)) = 1-(.2)^3 = .992.$

independence

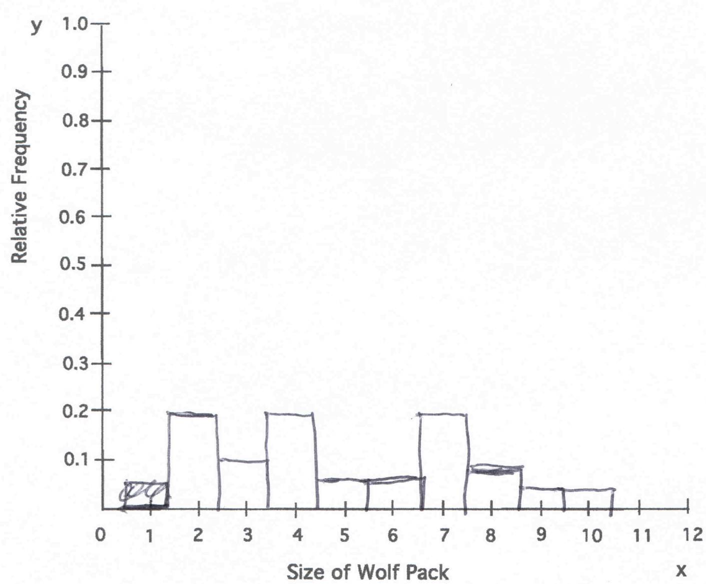
14, 10 points The following set of data on the number of wolves in winter wolf packs was collected in regions of Alaska, Minnesota, Michigan, Wisconsin, Canada, and Finland. The numbers recorded were:

6 10 7 5 7 7 2 4 3 2
2 3 9 4 4 2 8 7 8 4

(a) Make a table showing the outcomes(sizes of packs recorded), the frequencies and relative frequencies.

#	Freq.	Rel. Freq.
1	0	0
2	4	1/5
3	2	1/10
4	4	1/5
5	1	1/20
6	1	1/20
7	4	1/5
8	2	1/10
9	1	1/20
10	1	1/20
	20 = N	

(b) Draw a histogram of the data on the axes provided below.



15, 10 points In a carnival game, you pay $1 to play. Then you roll a die. If the uppermost face shows a 5 or a 6, the attendant gives you $2. If the uppermost face shows a 3 or a 4, the attendant gives you $1 and if the uppermost face shows a 1 or a 2, you give the attendant $1. Let X denote the earnings for this game.

(a) Show the probability distribution for the random variable X.

Dice	Earnings	Prob.
5 or 6.	\$1	$\frac{1}{3}$.
3 or 4	\$0	$\frac{1}{3}$.
1 or 2.	-\$2	$\frac{1}{3}$.

(b) What are the expected earnings for this game?

$$1 \cdot \frac{1}{3} + 0 \cdot \frac{1}{3} - 2 \cdot \frac{1}{3} = -\$\frac{1}{3}.$$

(c) If you play this game 100 times, How much (roughly) would you expect to win or lose?

$$-\frac{1}{3} \times 100 = -\$33 \text{ approx}$$